Modeling the Extended Warranty: A Managerial Focus

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We provide a model of an extended warranty. To maximize profit, a producer always wants to sell with some type of warranty as opposed to selling with no warranty. The extended warranty is more likely to be provided as the consumer becomes more patient, as the producer becomes impatient, or if the likelihood of product failure does not increase too much in the extended period. We show that there is a separating equilibrium in which the high quality producers sell with warranties and the low quality producers sell without warranties. We discuss the implications of warranty management in a government.

INTRODUCTION

Many products consumers and governments purchase come with some type of a warranty. An extended warranty, which extends the coverage of the warranty originally designed by the producer, is also available through the producer or a third party. The cost impact of extended warranties can be significant. A priori, it is unclear whether it is beneficial to the consumer or the producer to have an extended warranty. A warranty, similar to insurance, provides protection against bad outcomes and faulty manufacturing quality. The producer can use a warranty as a signal of the quality of the product. Hence, it may be that the extended warranty offered by the producer may very well be a worthwhile expenditure, but the same may not be true when the extended warranty is provided by a third party.

We develop a formal model to help understand what type of warranties are provided and consumed. We begin with the general model of insurance, benchmarking the model to be the best-case scenario for the consumer. The benchmark of the best-case scenario for the consumer means that there is neither hidden information nor hidden action; a buyer can purchase insurance in any scale of units, and lastly, all the competition is among the producers. In this situation, we show that the consumer will always want to fully protect herself against a bad outcome (i.e., full insurance). Of course, warranties are not identical to insurance and we cannot say with certainty that the consumer will still want the warranty under the new price and warranty pair. For example, warranties do not deal with partial warranty, partial replacement, or partial payment as insurance can. The producer replaces the entire unit if it fails to function as promised. The price of the standard warranty is built into the price of the final good consumed by the consumer. We transition our model to the warranty scenario, introducing the likelihood of success and failure of a product, the replacement cost, and a model for a standard warranty and an extended warranty.

We find that the producer will always want to sell with the standard warranty compared to no warranty, because it generates a higher profit. Higher profit is generated by the fact that there is risk-

sharing between the consumer and the producer. The decision increases in complexity when the extended warranty is introduced. There are situations in which the producer chooses to sell with only the standard warranty or only the extended warranty. The producer is more likely to sell with the extended warranty as the consumer becomes more patient, the producer becomes impatient, or the likelihood of failure in the later period has a small increase.

We extend the model to incorporate high-quality and low-quality producers, with the warranty used to signal the quality of the product. We find that the high quality producer will indeed sell with a warranty and will generate a positive profit. The low quality producer will not want to mimic the high quality seller and will not offer a warranty. Hence, a market separation occurs according to the quality of the product. The consumers are better off purchasing from the high quality producer and obtaining a positive expected utility. However, the results also suggest that the consumer may be better off consuming partially (via split bid procurement, etc.) from both types of producers in order to prevent the high quality producer from becoming a monopoly.

The outline of the paper is as follows. We first start with a brief discussion of warranties. In Section 2, we provide the literature review. In Section 3, we lay out the basic model and variables, and we provide our analysis in Section 4. Section 4 starts with an insurance scenario, and then we move on to a single-period warranty situation. We extend the model to two periods that have the possibility of extended warranties and analyze the situation of differing producer quality. In Section 5, we close by exploring some potential applications to government using the Department of Defense (DoD) as an example.

WARRANTY AND INSURANCE

In business and legal transactions, a warranty is defined as "an assurance by one party to the other party that specific facts or conditions are true or will happen" ("Warranty," n.d.). A warranty can actually cover a much broader range of issues than simply the replacement of a malfunctioning good during the coverage period. The warranty can be thought of as a promise to deliver a product. Warranty delivery can have quality, performance, time, and other aspects built into it. The costs of a warranty are incorporated into the production costs either ex ante or during production but not ex post of a sale. Therefore, the warranty is already factored into the final price of the good and the consumer does not pay an additional premium at the time of the sale. This means that the producer is effectively saying to the consumer, "We will sell you this product and this product will do this for this many years."

Insurance, on the other hand, is defined as "an equitable transfer of the risk of a loss, from one entity to another, in exchange for payment" ("Insurance," n.d.). Insurance is a state-contingent good or a policy. The purpose of insurance is to share risk between the policy holder and the provider for the price of a premium. This promise can be agreed upon ex ante, interim, or ex post of a sale and production. Of course, the state that insurance is contingent upon can cover all aspects of the promises made by a warranty. An insurance provider is essentially saying to the consumer, "If such and such happens, we will do this for you."

A standard warranty is built into the price of the good, while an extended warranty is an optional purchase. This optional purchase process can make an extended warranty's incentive and intent ambiguous. For example, a standard warranty may promise that an electronic good will work for one year. An extended warranty that covers that identical product for an additional year effectively claims that the electronic good will work for an additional year. Buying an extended warranty does not change the quality of the electronic good itself, given that the extended warranty is ex post of production. The extended warranty can be thought of as a risk-sharing mechanism between the producer and the consumer with information asymmetry given that the producer is much more informed about the failure rate than the consumer. In this aspect, an extended warranty cannot provide motivation or incentives to produce a higher-quality product. An extended warranty is conceptually similar to restricted insurance.

Another way to think about the difference between warranties and insurance is the following: A warranty deals with quality and function, while insurance deals with bad states of the world. A warranty could be seen as a subset of insurance. Insurance is used to deal with any bad outcome, such as rain

damage, fire, and so forth. However, a warranty only deals with a product's failure due to quality and function. For example, a laptop hard drive's motor failure after a month of normal usage would be covered under a warranty. A consumer can also buy insurance that replaces the hard drive when the hard drive fails due to a faulty motor. However, if the damage to the hard drive was caused by the laptop being dropped, then the damage would not be covered under the manufacturer's warranty, whereas it could be covered under an insurance policy. Formally, product failure can be stated or modeled as a *bad state of the world*. But not all bad states of the world can be modeled as product failures; therefore, the modeling of a warranty is studied as a subset of the insurance market.

LITERATURE REVIEW

The literature on warranty analysis is growing, especially in the area of extended warranties. Early research on warranties, such as Frees and Nam (1988); Blischke and Murthy (1994); Chen and Ross (1994); Mitra and Patankar (1997); Murthy, Iskandar, and Wilson (1995); and Nguyen and Murthy (1986), explored the cost aspects of the warranty from the perspective of a manufacturer. Other research, such as Lassar, Folkes, Grewal, and Costley (1998); Cooper and Ross (1985); and Lutz and Padmanabhan (1995), looked at the structure and potential behavioral implications of warranties in the manufacturer–consumer relationship. Complementing this research, DeCroix (1999) found a distinct signaling relationship between warranty, quality, and prices, an outcome supported in Balachander (2001).

In early research on extended warranties, Kelley and Conant (1991) explored the extended warranty's effect on consumers' attitudes on risk and manufacturers' perspectives on revenue and service. Padmanabhan (1995) and Lutz and Padmanabhan (1998) found that different segments of the market, based on usage and valuation, make it optimal for a manufacturer to provide a menu of extended warranty contracts. Likewise, Mitra and Patankar (1997) found that extended warranties are advantageous for the manufacturers. Lam and Lam (2001) explored the optimal actions of consumers, as well as manufacturers, when faced with extended warranties. Thomas and Rao (1999) and Murthy and Djamaludin (2002) performed extensive reviews of the early research on warranties in the operations and production literature.

More recently, the literature has focused on the design of extended warranty contracts, as in Jack and Murthy (2007); Hartman and Laksana (2009); Li, Mallik, and Chhajed (2012); and Heese (2011), as well as the reasons behind consumers choosing to buy extended warranties, as in Chen, Kalra, and Sun (2009) and Jiang and Zhang (2011). Chen et al. (2009) argued that extended warranties must be overpriced in order to compensate for the standard warranty time period. Desai and Padmanabhan (2004) looked at the warranty as a type of insurance for risk-averse customers.

The literature discussed places a heavy emphasis on analyzing the failure rate or deriving the cost of a unit based on the type of failure (single versus multiple dimension, etc.) and failure rates. These types of modeling are excellent at providing the optimal price to charge given the quality (i.e., pre- to post-production, but before the product goes on sale), but the models do not explore how other producers and consumers will best respond. Behavioral results, while extremely important and informative, may fall short when it comes to setting a benchmark for behaviors and expectations.

Our contribution adds to the literature by formally modeling the warranty situation via economic theory and allowing for the strategic interaction between the consumers and the producers. We formalize our economic environment by generalizing the industrial organization literature (Shy, 1995). This form of modeling provides the benefit of showing how the consumers and different types of producers (high quality and low quality) should respond to one another and what type of warranty (extended warranty, standard warranty, or no warranty) should be provided.

MODEL

We first begin the model by defining the consumers and the producers. The consumer of the good, or the department of defense (DoD) in our model, labeled as *D,* is a von Neumann-Morgenstern expected

utility maximizer. The consumer values the good purchased at $V > 0$. There are two types of producers of goods in the market: S_q where $q \in \{h,l\}$ for high-quality type and low-quality type sellers. The producers can be thought of as contractors who are providing goods to the DoD. High-type producers are more likely to produce a product of a higher quality as we will discuss further in this section.

The sellers compete in a Bertrand setting, meaning that they compete over price rather than quantity, and are profit maximizers. There is only a single good being produced. This single good can be sold with *x* type of warranty. For our model, we focus on the following types of warranty: w= standard warranty, ew = extended warranty, and *nw* = no warranty. For simplicity, the extended warranty doubles the standard warranty. Therefore, even though there is only a single good, in effect, there are three potential goods being sold. We assume that if the product failure is covered by the warranty, the producer will provide the consumer with a new product.

There are two potential states of the world: product failure and product success. We denote the cumulative distribution function (CDF) of a good not needing a warranty service when it is produced with effort level e by a producer of type t, and warranty service of type w is $F(e \mid S_q, x)$, and $F(e \mid S_h, x) \le F(e \mid S_l, x)$ for all e. This leads to the probability of a product produced by a high-type seller not needing a warranty service first order stochastically dominates that of a low-type seller. For the scope of this paper, we project the effort levels to a single effort level, and the probability of not needing warranty service (success) for type *q* providing *x* warranty is π_q^x, and $\pi_h^x < \pi_l^x$. We index the probability with time period $t \in \{1,2\}$. In the most general scenario, the probability of success is denoted as $\pi^x{}_{q,t}$. Probability of success in the second period is lower than the probability of success in the first period: $\pi^x{}_{q,1} < \pi^x{}_{q,2}$. We assume that the reliability or the product quality is exogenous of the warranty type and the probability of failure is independent: $\pi^x{}_{q,t} = \pi^y{}_{q,u}$ for $x \neq y$. Future extension can be that the quality is endogenous. We will drop the superscripts or subscripts when not needed.

The cost of producing the good for both producers is $0 < c < \pi_1 V$, meaning the consumer's expected value of the good is more than the cost of production. Otherwise, there is no reason for the market to exist. Furthermore, there is room for gains from trade in this model. The price charged for the product with warranty type *x* is p^x. In the multi-period setting, let δ_D and δ_P be the discount factors for the consumer and the producer, respectively. Discount factors are $\delta \in (0,1)$.

ANALYSIS

We begin the analysis by looking at the most general case, the case of insurance. The purpose of insurance is to protect the consumer against any bad states of the world. Insurance can be written to protect the consumer against failure of the product that is due to the producer, as well as failure due to nature (fire, earthquake) or the end-user (misuse or careless use of the product). Furthermore, a warranty is a discrete service while insurance can be continuous.

Let us first consider the first-best case scenario—with flexibility, with perfect competition among sellers, and without any information asymmetry. The general insurance case with two possible states of the world (good and bad), the consumer's utility function is

$$U_D = \begin{cases} U(V - aV + I - bI) & \text{in bad state} \\ U(V - bI) & \text{in good state} \end{cases} \quad (1)$$

where $V > 0$ is the value of the product to the consumer, $a \in [0,1]$ is the portion of value lost in the bad state, I is the insurance payout, and $b \in [0,1]$ is the cost of insurance. The probability that the consumer is in a good state is $\pi \in (0,1)$. Therefore, the consumer's expected utility function is

$$EU_D = \pi U_D(V - bI) + (1-\pi)U_D(V - aV + I - bI). \tag{2}$$

The consumer's optimization problem is to determine the optimal insurance level, *I*. The first order condition provides that

$$\frac{\pi}{(1-\pi)} = \left(\frac{\partial U_D(V - aV + I - bI)}{\partial I} \Big/ \frac{\partial U_D(V - bI)}{\partial I} \right)\left(\frac{1-b}{b} \right). \tag{3}$$

To explore the insurer's optimization problem, we have to consider the insurer's profit in a good state and a bad state of the world:

$$\text{Profit} = \begin{cases} bI - I & \text{in bad state} \\ bI & \text{in good state} \end{cases}. \tag{4}$$

Therefore, the insurer's expected profit is $E(\text{Profit}) = \pi bI + (1-\pi)(bI - I)$. In a competitive market with information symmetry, the expected profit is zero; therefore, the unit price of insurance *b* equals the probability of a bad state: $b = (1-\pi)$.

Finally, putting the consumer and the insurer's optimization problem together, we obtain $\frac{1-b}{b} = \left(\frac{\partial U_D(V - aV + I - bI)}{\partial I} \Big/ \frac{\partial U_D(V - bI)}{\partial I} \right)\left(\frac{1-b}{b} \right)$, and finally, the equilibrium condition is $\frac{\partial U_D(V - bI)}{\partial I} = \frac{\partial U_D(V - aV + I - bI)}{\partial I}$. With the usual assumption of an expected utility function, this equality holds when $V - bI = V - aV + I - bI$, which suggests a full insurance of $I = aV$. This brings us to our first important result.

Proposition 1: Under the insurance setting with complete information and perfect competition among the insurance providers, the consumer would like to be fully insured against the bad state and consume exactly the same in both the good state and bad state of the world.

Of course, this setting was the first best scenario, meaning, this is the best that the consumer can possibly do. We start by breaking down some of the assumptions in the first best scenario in the subsections to follow and determine whether a warranty is still desired by the consumer and the producer. Here are some technical reasons why warranty settings need additional analysis. First, we cannot partially insure the consumer and must replace the good whether it is a partial or full damage, therefore $I = V$. Given the differences between warranties and insurance, the cost is initially born by the producer (whether it is passed on to the consumer or not). Third, a warranty can be used as a signal of the quality of a good. Lastly, the warranty is provided by the producer, and not a third-party insurance company.[1]

Next, we break down the warranty setting and show that a warranty is still desirable for the consumer and the producer. Initially, we consider the case with complete information (just like the insurance case) by a single producer and a single consumer. The reason we start with a single producer is evident in the next section. If there is only one producer, the producer is a monopoly and will extract the entire surplus from the consumer.

Warranty (Single Period)

The potential surplus to extract from the consumer depends on whether the warranty is provided or not. The expected utility in both cases is

$$\text{EU} = \begin{cases} U(V - p^{w}) & \text{Warranty} \\ \pi U(V - p^{nw}) + (1-\pi)U(-p^{nw}) & \text{No warranty} \end{cases}. \tag{5}$$

The producer finds the proper price to charge in each case for the expected utility to equal zero. When a warranty is provided, because $U(V, p)$ is a continuous increasing function in V and a decreasing function in p, we can always find a p^{w*} such that $U(V - p^{w*}) = 0$ within an affine transformation.[2] When the warranty is not provided, the producer charges p^{nw*} such that $\frac{\pi}{1-\pi} = -\frac{U(-p^{nw*})}{U(V - p^{nw*})}$. The solution for this equality, p^{nw*}, exists for the same reason as with the case in which a warranty is provided.

The producer's problem is as follows: Since the producer is a monopoly in this setting, the producer will compare the expected profit (expected profit = price – expected cost[3]) under both scenarios and take the best course of action. In short, the expected profit in both cases is

$$\text{Expected Profit} = \begin{cases} p^{w*} - c/\pi & \text{Warranty} \\ p^{nw*} - c & \text{No warranty} \end{cases}. \tag{6}$$

Lemma 1: The producer will sell the good with a warranty if $\pi p^{w*} \geq p^{nw*}$.

Notice that, unlike the insurance case, there will be cases in which the seller provides no warranty. The producer will sell with the warranty if the price the producer can charge with a warranty multiplied by the probability of a good state is higher than the price the producer can charge without a warranty. So simply being able to charge more in a warranty situation is not enough to sell the product with a warranty. The benefit has to be high enough that, even when it is contracted by the probability of a good event, it is still higher than the price the producer can charge without a warranty. Placing this in context, as the good is less likely to fail (π increases), the producer is more likely to provide the good with a warranty, that is, either the price the producer can charge has to be very high or the quality of the product has to be very high.[4]

We illustrate our result using two examples: a linear utility function (risk-neutral consumer who simply wants to maximize the value consumed) and a constant absolute risk-aversion utility function (risk-averse consumer).

Example: Linear Utility Function

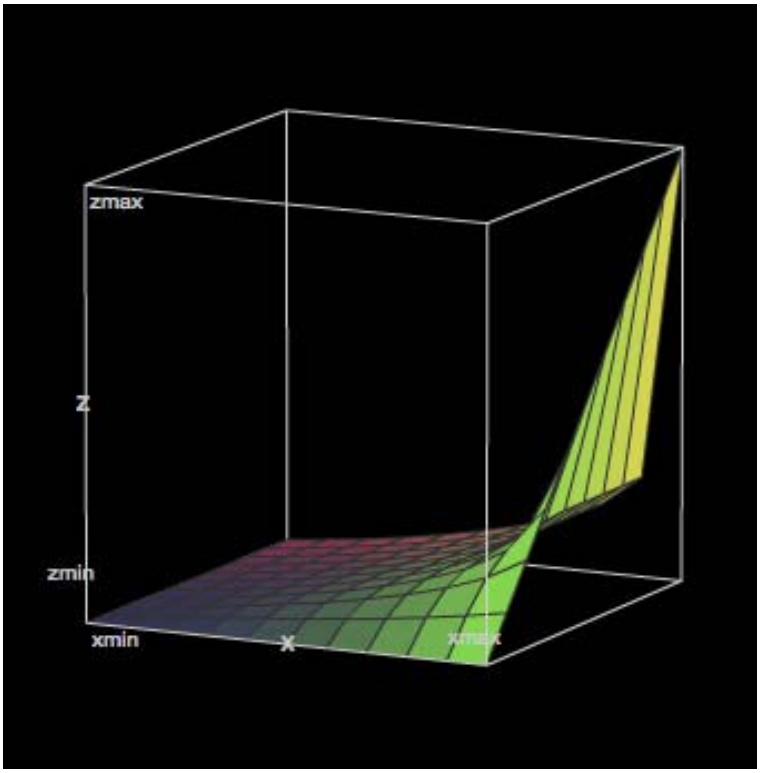
Suppose the consumer has the following expected utility function: $U(V, p) = V - p$. This model of utility function is useful when we are looking to maximize the total value or dollar. We find that the optimal price to charge under the warranty and no-warranty scenario is as follows: $p^{w*} = V$ and $p^{nw*} = \pi V$. Because $p^{w*} \geq p^{nw*}$, as stated in Proposition 2, the producer will always sell the good with a warranty. An explanation of why this is the case, is that the consumer is indifferent between the two prices and the warranty pair with an expected utility of 0. Therefore, we only need to show that the profit of the producer is higher in the warranty setting than the no-warranty setting. The profit in the no-

warranty setting is $\pi V - c$, which is only a fraction (since $\pi < 1$) of the profit in the warranty setting of $V - c/\pi$. Hence, a warranty will always be provided in this risk-neutral scenario.[5]

Example: Constant Absolute Risk-Aversion Utility Function

Suppose the consumer has the following expected utility function: $U(V, p) = 1 - \exp(-\gamma(V - p))$ where γ is the risk-aversion coefficient. This problem is not as trivial as the linear utility function example. The optimal price to charge in order to extract all the surplus when providing the good with a warranty is still $p^{w^*} = V$, regardless of the level of risk aversion. The optimal price to charge when the warranty is not being provided is found by finding a price that satisfies $\frac{\pi}{1-\pi} = -\frac{1-\exp(\gamma p^{nw^*})}{1-\exp(-\gamma V + \gamma p^{nw^*})}$. The closed form solution to the condition is $p^{nw^*} = -\left(\frac{1}{\gamma}\right)\ln\left((1-\pi) + \pi\exp(-\gamma V)\right)$. This solution states that, as the probability of failure decreases, the p^{nw^*} will approach p^{w^*}, charging the consumer the exact value of the good and as the good is more likely to become faulty, p^{nw^*} will approach 0. Furthermore, as the value of the good increases, the price of the good, p^{nw^*}, also increases, but it is not in a linear fashion as it was in the previous example. Figure 2 provides a graphical representation for the intuition. Just as in the linear utility case, we find that the producer will always sell with a warranty and obtain a higher profit.

FIGURE 1
p^{nw^*} UNDER CONSTANT ABSOLUTE RISK-AVERSION UTILITY FUNCTION



This figure depicts the optimal price to charge under the no-warranty setting given a risk-averse consumer. $U(V,p)=1-\exp(-\gamma(V-p))$: calibration are $V=10$, risk aversion corresponding to the y-axis, π corresponding to the x-axis, and p^{nw*} corresponding to the z-axis.

We conclude this section with this important result. The producers are providing a warranty, not because it is better for the consumer but because it maximizes their profit. The producers earn more profit by providing a warranty compared to not providing any warranty, because they capture the surplus of providing protection against the bad state (failure of the product).

Proposition 2: With a risk-neutral and a risk-averse consumer, the producer always earns a higher profit by including a warranty.

Proof: We begin by normalizing the expected utility to $U(0)=0$. The expected value under the no-warranty case is $\pi V - p^{nw*}$. The certainty equivalent of obtaining the expected utility of zero under the no-warranty case is exactly when the consumer consumes zero. The expected utility function is concave for the risk-averse consumer. Then the expected value under the no-warranty case must be greater than zero. Then by Lemma 1, the producer will always sell the good with a warranty.

Standard Warranty versus Extended Warranty

Allowing for an extended warranty requires another set of possible warranty–price pairs. Furthermore, it requires us to extend the single-period model to a two-period model. The expected utility of the consumer is given by

$$\text{EU}=\begin{cases} & \\ U(V-p^{ew})+\delta_D U(V) & \text{Extended warranty} \\ U(V-p^{w})+\delta_D\left(\pi_2 U(V)+(1-\pi_2)U(0)\right) & \text{Warranty (single period)} \\ \pi_1 U(V-p^{nw})+(1-\pi_1)U(-p^{nw})+\delta_D\left(\pi_2 U(V)+(1-\pi_2)U(0)\right) & \text{No warranty} \end{cases} \tag{7}$$

where the latter parts are the discounted expected utility in the second period. The expected profit of the producer is given by

$$\text{Expected Profit}=\begin{cases} p^{ew*}-c/\pi_1-\delta_P\left(c/\pi_2-c\right) & \text{Extended warranty} \\ p^{w*}-c/\pi_1 & \text{Warranty} \\ p^{nw*}-c & \text{No warranty} \end{cases} \tag{8}$$

Just as in the single warranty section, the producer has to determine the proper price in order to extract the entire surplus: $EU=0$.

First, we determine the difference between a warranty and an extended warranty. The proper price to charge with an extended warranty p^{ew*} is the solution to $\delta_D=-\dfrac{U(V-p)}{U(V)}$. The proper price to charge with the standard warranty p^{w*} is the solution to $\pi_2\delta_D=-\dfrac{U(V-p)}{U(V)}$. We analyze this in the linear utility setting as before $U(V,p)=V-p$. The optimal prices to charge are $p^{ew*}=V(1+\delta_D)$ and $p^{w*}=V(1+\delta_D\pi_2)$. The profit in each case is $V(1+\delta_D)-c/\pi_1-\delta_P\left(c/\pi_2-c\right)$ and $V(1+\delta_D\pi_2)-c/\pi_1$ for

the extended warranty and the standard warranty, respectively. When the producer provides an extended warranty, the differences are positive. The difference in profit becomes

$$V(1+\delta_D) - c/\pi_1 - \delta_P\left(c/\pi_2 - c\right) - V(1+\delta_D\pi_2) - c/\pi_1 = (1-\pi_2)\left(\delta_D V - \delta_P\, c/\pi_2\right) = \Delta\text{Prof(ew-w)}. \quad (9)$$

The difference in profit, ΔProf(ew-w), is positive when $\delta_D V \geq \delta_P c/\pi_2$. If so, the extended warranty will be sold and the producer will generate a higher profit compared to the standard warranty. On the flip side, if π_2 is low, meaning it has a high probability of being faulty in the second period, then the extended warranty will not be provided and only the standard warranty will be provided. Holding the success rate π_2 fixed, the discount factor plays a role in determining whether the producer goes with the extended warranty plan or the standard warranty plan. If the discount factor is rather low (discount rate is high) for the producer, then the producer will be more forgiving of the lower success rate at Period 2. In short, a decrease in the discount factor for the producer (δ_P), an increase in the success rate in Period 2 (π_2), or an increase in the discount factor for the consumer (δ_D) all attribute to an increase in the likelihood of the extended warranty being provided. This result is summarized as Lemma 2 and Lemma 2.1.

Lemma 2: When maximizing the total value (linear expected utility) and comparing the extended warranty plan and the standard warranty plan, the producer will provide the extended warranty if $\delta_D V \geq \delta_P c/\pi_2$. Otherwise, the producer will provide the standard warranty.

Lemma 2.1: When maximizing the total value (linear expected utility) and comparing the extended warranty plan and the standard warranty plan, the likelihood of providing the extended warranty increases as a function of δ_D and π_2 while decreasing in δ_P.

Standard Warranty versus No Warranty (2-Period Setting)

The steps taken to analyze the difference between the standard warranty scenario and the no-warranty scenario are similar to those described in the warranty setting. Continuing with the linear expected utility function of simply maximizing the expected value, the optimal price to charge under the warranty scenario is $p^{w*} = V(1+\delta_D\pi_2)$, generating a profit of $V(1+\delta_D\pi_2) - c/\pi_1$. For the no-warranty scenario, the optimal price to charge is $p^{nw*} = V(\pi_1 + \delta_D\pi_2)$, generating a profit of $V(\pi_1+\delta_D\pi_2) - c$. The difference in profit can be simplified to $\Delta\text{Prof(w-nw)} = (1-\pi_1)V - (1-\pi_1)(c/\pi_1)$, which is always positive. The result is that the producer will always choose to sell with a warranty compared to selling without a warranty.

Lemma 3: When maximizing the total value (linear expected utility) and comparing the standard warranty plan to the no-warranty plan in the two-period setting, the producer will always sell with the standard warranty, independent of the discount factor.

Extended Warranty versus No Warranty

We evaluate the difference between the extended warranty scenario and the no-warranty scenario. We only need to compare the situation in which $\delta_D V < \delta_P c/\pi_2$. This is the case because if $\delta_D V \geq \delta_P c/\pi_2$, then we know from Lemma 2 that the producer will choose to provide the extended warranty over the standard warranty, and by Lemma 3, that the producer always prefers to sell with a warranty as opposed to no warranty.

The difference in profit for these two cases simplifies to

$$\Delta\text{Prof(ew-nw)} = (1-\pi_1)(V - c/\pi_1) + (1-\pi_2)(\delta_D V - \delta_p c/\pi_2),$$

and the producer will decide to sell with the extended warranty versus no warranty if the difference is positive. Unfortunately, because the first-period effect is positive and the second-period effect is negative, we cannot state definitely which of the two options the producer will choose for all ranges of value. If the expected profit from the first period is much higher than the expected discounted loss in the second period, the producer will choose to sell with an extended warranty compared to not selling with any warranty. It is important to note that, in the case in which $\delta_D V < \delta_P c/\pi_2$, both the extended warranty and the no-warranty scenarios are dominated by the standard warranty scenario, leading to an outcome that determination of which of the two settings offer the higher profit may be unnecessary. These analyses provide us with the following main result.

Proposition 3: Consider the case of the two-period model with its ability to provide no warranty, a standard warranty, and an extended warranty. If $\delta_D V \geq \delta_P c/\pi_2$, then the producer's profit is the highest when providing a product with the extended warranty, second highest when providing the standard warranty, and the lowest when providing no warranty. Therefore, the producer will sell with an extended warranty. If $\delta_D V \geq \delta_P c/\pi_2$ is not observed then the producer's profit is the highest when selling with the standard warranty and the producer will not sell the extended warranty or opt for no warranty.

In summary, this proposition states that the producer will always choose to sell with some type of warranty whether it is an extended warranty or a standard warranty. Deciding between the standard warranty and the extended warranty occurs due to several factors. In particular, how much does the success rate of a product decrease when it transitions from Period 1 to Period 2. If this decrease is high, meaning the product has a short lifespan, the extended warranty will not be provided. In addition, the discount factor that the producer and the consumer place on the second period's consumption will impact the decision. The more patient the consumer is, or the less patient the producer is, the more likely it is that the extended warranty will be provided.

Incomplete Information

We started out the complete information case with only one producer. We now introduce two producers and show that the market will collapse to a single producer. We only focus on a single-period model with the standard warranty and no warranty for intuition. However, we introduce two different types of sellers: high quality and low quality.

There are four possible situations we need to consider. In Case 1, both producer types produce without a warranty. In Case 2, both producer types produce with a warranty. In Case 3, the low type produces with a warranty while the high type produces without a warranty. In Case 4, the high type produces with a warranty while the low type produces without a warranty.

Case 1: Neither Producer Type Offer a Warranty

If both producers produce without a warranty, then there is no way for the consumer to distinguish the difference between the two types of producers. Since the producers are in a Bertrand competition, they compete over price until price equals marginal cost, *c*. This leads to an expected profit of $p^{nw} - c$ for both producers and ends with zero profit. The expected utility for the consumer is $\pi U(V - p^{nw}) - (1-\pi)U(-p^{nw})$ where $\pi = (\pi_l + \pi_h)/2$. The consumer retains the entire surplus in this market.

Case 2: Both Producer Type Offer a Warranty

If both producers produce with warranties, the consumer still cannot distinguish between the two types of sellers. In a Bertrand competition the two producers compete over price. However, the Nash equilibrium in this case is different from Case 1. This is because it is cheaper for the high quality producer to provide a warranty compared to the low quality producer. The high quality producer can lower the

price with a warranty to $p^w = c/\pi_l - \varepsilon$ for $\varepsilon > 0$. If the low quality producer charges anything less than c/π_l, the low quality producer obtains a negative profit. Therefore, in Case 2, the warranty is provided by the high quality producer while the low quality producer cannot mimic the high quality producer without losing profit. The high quality producer earns a strictly positive profit because $p^w - c = c/\pi_l - \varepsilon - c > 0$.

Case 3: High Quality Producer Without a Warranty and Low Quality Producer With a Warranty

In case 3, we check whether it is rational for the high quality producer to produce without a warranty and for the low quality producer to produce with a warranty. This is rational if neither of the producers have an incentive to deviate from this strategy. If the high quality producer is providing no warranty, then the lowest price that the producer can possibly charge is $p^{nw} = c$, which provides a profit of zero. The lowest price the low quality producer can charge is $p^w = c/\pi_l > p^{nw}$, which also provides a profit of zero. We know from Case 2 that the high quality producer would mimic the low quality producer and sell with a warranty to obtain a positive profit. Therefore, this strategy cannot be sustained in equilibrium.

Case 4: High-Type Producer with Warranty and Low Type without Warranty

The final case in our analysis is the case of a separating equilibrium in which the high quality producer offers a warranty while the low quality producer offers no warranty (or exits the market). The low quality producer can charge a price ranging from $p^{nw} = [c, p^w - \varepsilon]$ for $\varepsilon \to 0$, which provides a profit ranging from $[0, p^w - c]$. The lowest price that the high quality producer can charge is $p^w = [c/\pi_h, c/\pi_l - \varepsilon]$ for $\varepsilon \to 0$ and this provides a profit ranging from $[0, c/\pi_l - c/\pi_h]$ to $c/\pi_l - c/\pi_h > 0$. Consider the extreme situation in which the low quality producer charges the lowest possible price and the high quality producer charges the highest possible price. Because lowering the high quality producer's price will only decrease the profit, the producer has no incentive to deviate. The low quality producer cannot increase the price because the low quality producer is already earning zero profit. This equilibrium is a sustainable strategy by the producers, assuming that the consumer will choose to buy from the high quality producers over the low quality producers.

We now compare the expected utility of the consumer to determine if the consumer will purchase from the high quality producer with a warranty as opposed to the low quality producer without a warranty. The expected utility with a warranty is $U_D(V - p^w) = U_D(V - c/\pi_l)$ and the expected utility without a warranty is $\pi_l U_D(V - p^{nw}) + (1-\pi_l)U_D(-p^{nw}) = \pi_l U_D(V - c) + (1-\pi_l)U_D(-c)$. The expected value that the consumer obtains without the warranty is given by $\pi_l(V - c) + (1-\pi_l)(-c)$, which is less than the expected value from consuming with warranty $V - c/\pi_l$. Because the expected utility is a concave function, it must be that the utility of consuming $V - c/\pi_l$ with certainty is higher than the expected utility of consuming $\pi_l(V - c) + (1-\pi_l)(-c)$ in expectation. This means that the consumer simply gets more value with certainty when purchasing the good with a warranty compared to the expected value obtained by purchasing the good without a warranty from the low quality consumer.

Proposition 4: In the game of incomplete information, there is a separating equilibrium in which the high quality producer sells with a warranty and the low quality producer is driven out of the market.

The outcome is the high quality producer will sell with a warranty and obtain a profit, the low quality producer will sell without a warranty and make no profit, and the consumer will purchase from the high quality producer with a warranty and obtain a positive utility. This result suggests that consuming partly from the low quality producer instead of driving it out of business (collapsing the market to a single

producer) may be beneficial because it creates competition among the producers. The consumer generates a higher expected utility in the two producer cases compared to the single producer case in our analysis.

POTENTIAL BENEFITS OF EXTENDED WARRANTIES TO THE DoD

As shown in the analysis, the customer (in our case DoD) would always prefer a warranty. This outcome introduces important managerial considerations. Although we do not address, in this paper, all of the issues mentioned heretofore, warranty management is indeed important and does deserve further investigation. Warranty management is a challenge that many of today's supply chains must confront, especially the DoD. Warranties involve customers returning allegedly defective units to the supplier in return for a replacement unit or monetary credit. Warranties involve many complex interactions, such as probabilistic repairs, high demand rates, multiple sources of supply, and strict customer service constraints. Though it is an expensive and complex logistical operation, many organizations mismanage or ignore their warranty policies, resulting in inefficiencies throughout the supply chain and ill will among supply chain partners.

Extended warranties, sometimes known as service contracts, negotiated warranties, or extended service contracts, may be offered or requested by the DoD when it is purchasing new items or negotiating new contracts. Extended warranties protect the DoD against breakage after the normal warranty period has expired. In the DoD, extended warranties can be especially important for repairable items. Product Quality Deficiency Reports (PQDRs) capture whether an item is under warranty or not and include the expiration of the warranty. Products still under warranty are treated differently than products out of warranty.

Warranties can be considered a type of insurance that protects the DoD from manufacturing and material defects. The premiums of standard warranties are typically built into the negotiated purchase price of a product. If a product is found to be defective and covered under warranty, the DoD receives a replacement product (or sometimes a monetary credit) according to the contract.

When a supplier and the DoD enter into a purchasing agreement, the warranty and extended warranty terms must be agreed upon in the negotiation phase of the acquisition process. Both parties must carefully manage the terms of the warranty portion of the contract, as many contracts can cover many years. The aspects of warranties that should be considered include (but are not limited to) the following: what constitutes a defect; what is agreed upon to be "standard usage" of the product (which defines the conditions under which an item is covered under warranty); what is the standard operating procedure for claiming a warranty; whether or not refurbished items can be supplied as warranty replacements; what timeline the supplier agrees to supply a replacement item; whether or not the DoD can request a monetary credit in lieu of a replacement; and what are the monetary repercussions to either party's failure to follow the agreed upon contract.

An extended warranty typically extends the amount of time after purchase that a product remains under warranty. However, it may also cover a wider breadth of product failures than a standard warranty. For example, some extended warranties cover accidental breakage or water damage. The DoD must pay an extra premium in order to receive coverage, and the timing and requirements of replacement may change from the standard warranty agreement.

Suppliers are not the only entities that can offer extended warranties. Two other options exist. First, a third-party service provider may offer extended warranties to the DoD. Second, the DoD may choose to "self-insure" its purchases. This would entail budgeting money to cover repairs rather than paying the premium to suppliers or a third party to cover repairs outside of the standard warranty period.

There are many reasons that the DoD would want to buy extended warranties on certain products. This could include the reduction of the depot level and field level repair workload for DoD employees, risk aversion to loss of uptime resulting from defective parts, and protection against suppliers abandoning a product in favor of new technology. An extended warranty may be advantageous when using a product with a high probability of breakage or when the DoD is a heavy user of a particular product. In addition, a long-term contract may incentivize the DoD to enter into a long-term extended warranty contract.

Although there are positive aspects of an extended warranty, there are some negative aspects. The time and effort required to receive a replacement item or credit may be extensive and highly variable. This could force the DoD into additional carrying costs associated with holding more inventory than would otherwise be necessary to protect itself against long and erratic lead times. Suppliers may scrutinize and falsely reject warranty claims, resulting again in long lead times and potential legal ramification (along with their associated costs). Furthermore, the premium charged by the extended warranty provider may be too high compared to the benefits gained from having this type of extra insurance.

CONCLUSION

This paper provides a formal model of an extended warranty. We find that the producer will always want to sell with some type of warranty compared to no warranty in order to maximize profit. The extended warranty is more likely to be provided as the consumer becomes more patient, the producer becomes impatient, or the likelihood of the product failure does not increase too much in the extended period. Finally, we show that there is a separating equilibrium in which the high quality producers will sell with a warranty and the low quality producers will sell without a warranty, with the consumer purchasing from the high quality producer. These results also suggest that the consumer, or the DoD, may want to consume partly from the low-quality producer (via split bid procurement, etc.) in order to keep the competition between the producers. If there is only a single producer, the producer is able to extract more value from the consumer and generate a higher profit. The consumer generates a higher value when there are two producers, compared to having only one producer, as in our analysis.

There is much room for future research when it comes to allowing for strategic interactions with extended warranties. First, as stated in the modeling process, we can allow for the quality of the product to depend on the type of warranty selected and the amount of effort exerted to produce the product. This is a necessary step to understand how the consumer can use a warranty and an extended warranty as an incentive to motivate the producer.

The producer can use extended warranties as a method of price discrimination. Producers can create a different type of warranty and price pair to target consumers with different risk and discount rates. This could be why we see both the extended warranty (at an additional cost) and the standard warranty being offered. The less risk-averse and impatient consumer will elect to consume with the standard warranty, while the more risk-averse and patient consumer will elect to go with the extended warranty.

The timing of the creation of the warranty contract plays a significant role in determining the quality and the cost of the product. The warranty contract is written ex post of the product development in the current literature. However, especially for the DoD, the consumer may demand a particular good and warranty, and is in a position to make such a demand. Therefore, specifying a particular warranty contract ex ante of the product development affects not only the quality, but also the cost of production. It is not clear whether extended warranties ought to be requested or how this change in the warranty contract timing will change the behavior of the consumer and the producer.

ENDNOTES

1. While it is true that there are third-party extended warranty providers, this is outside the scope of this paper.
2. Proof is available from authors.
3. Derivation for the cost is omitted. The warranty case follows the logic of geometric sum: $c + (1-\pi)c + (1-\pi)^2 c + \ldots = c/\pi$.
4. An excellent comparative statics is a change in V and change in risk aversion.
5. Of course, there is an implied assumption that profit is greater than zero. This is true as long as $V > c/\pi$. Otherwise, the producer will simply exit the market.

ACKNOWLEDGEMENTS

We would like to thank Dr. John Khawam for his technical assistance and helpful comments. The information and opinions presented here are that of the authors and do not represent the United States Government or the Department of Defense.

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