Understanding CVA, DVA, and FVA: Examples of Interest Rate Swap Valuation

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Financial statements of major money-center commercial banks increasingly include reference to a credit valuation adjustment (CVA), debit (or debt) valuation adjustment (DVA), and funding valuation adjustment (FVA). This article explains the concepts behind CVA, DVA, and FVA using examples of interest rate swap valuation. A binomial forward rate tree model is used to get the value of the swap assuming no default. The CVA (the credit risk of the counterparty) and the DVA (the credit risk of the entity itself) depend on assumptions about the probability of default, the recovery rate and the expected exposure, which depends of projected values and settlement payments for the swap. The FVA arises when an uncollateralized swap is hedged with a collateralized or centrally cleared contract. In this version of the paper, two methods to calculate FVA are shown, both using the same assumptions about the credit risk parameters for the bank.

INTRODUCTION

Financial statement analysis of money-center banks nowadays requires an understanding of CVA, DVA, and FVA, sometimes referred to collectively as the "XVA." These are the acronyms for Credit Valuation Adjustment, Debit (or Debt) Valuation Adjustment, and Funding Valuation Adjustment. For example, JP Morgan Chase's Corporate and Investment Bank includes this bullet point for 4th quarter 2014 financial performance: "Credit Adjustments & Other loss of $452mm driven by net CVA losses, as well as refinements to net FVA/DVA valuation." Bank of America reports: "FVA losses were $497 million for the three months ended December 31, 2014 resulting from a one-time charge related to the adoption of funding valuation adjustments related to uncollateralized derivatives in the company's Global Markets business."

This paper explains the concepts behind CVA, DVA, and FVA with examples of interest rate swap valuation. CVA is the least controversial of these adjustments. The idea is that the value of a financial asset such as a purchased option contract is the value assuming no default (here denoted VND) less the CVA:

$$\text{Value}^{\text{Asset}} = \text{VND} - \text{CVA} \tag{1}$$

The VND is the present value of the projected future cash flows using discount factors that reflect "risk-free" interest rates. These rates are not necessarily yields on government bonds, which typically are lower than desired. Treasuries are the most liquid securities in the market, are in demand for use as collateral, and have state and local tax exemptions. The ideal discount factors would come from securities that have the same liquidity and tax status as the ones under consideration but default risk that approaches zero. Before the financial crisis of 2007-09 banks used discount factors derived from LIBOR-based time deposits, futures contracts, and interest rate swaps. In effect, the LIBOR curve was the proxy for bank "risk-free" rates and the stability in the spread between LIBOR and Treasury bills (i.e., the TED spread) justified its usage.

The financial crisis exposed the liquidity and credit risks embodied in LIBOR as the TED spread spiked upward, reaching a pinnacle in September 2008, and remained well above pre-crisis levels. Banks have since adopted rates on overnight indexed swaps (OIS) as the new proxy for "risk-free" status. These derivatives are based on the total return on a reference rate that is compounded daily. In the U.S., this is the effective fed funds rate, the weighted average of overnight inter-bank rates on borrowing and lending deposits at the Federal Reserve Bank. OIS discounting has become the norm to obtain values for derivatives before consideration of counterparty credit risk; see Smith (2013) and Hull-White (2013) for further details.

A credit risk model is used to get the CVA. The inputs to the calculation include the expected exposure to default loss (i.e., the expected future values and settlement payments), the probability of default by the counterparty, and the recovery rate if default were to occur. An advantage to modeling the VND and CVA separately is that the benchmark rates that drive the former capture *macroeconomic* factors such as expected inflation, the business cycle, and monetary and fiscal policy, whereas the latter reflects mostly *microeconomic* factors specific to the counterparty. Also, the rules to get preferable accounting treatment require identification as to whether the hedge is for changes in benchmark interest rates (the VND) or for changes in counterparty credit risk (the CVA); see Gastineau, Smith, and Todd (2001).

DVA is the flip side to CVA. To the option writer, the value of its liability is VND minus the DVA:

$$\text{Value}^{\text{Liability}} = -\,(\text{VND} - \text{DVA}) \tag{2}$$

The minus sign before the expression indicates a liability. The same parameters are used to estimate DVA as CVA. In principle, CVA = DVA for a derivative that has *unilateral* credit risk such as an option contract. The difference is only in perspective—CVA is the credit risk facing the option holder whereas DVA reflects the credit risk of the entity that writes the contract.

When the credit risk of the option writer goes up due to a higher estimated probability of default, or perhaps a lower assumed recovery rate if default were to occur, it is reasonable for the option holder to recognize a lower valuation due to a higher CVA. In fact, to not do so would be suspect. However, recognition of a gain by the option writer as its credit quality deteriorates and the value of its liability goes down due to the higher DVA is quite controversial. Lehman Brothers notoriously posted huge DVA gains in the days before its bankruptcy in September 2008. The Basel Committee on Banking Supervision since then has ruled that gains from a higher DVA do not count as increases in Tier I equity for measuring capital adequacy in commercial banks.

Interest rate swaps, and forward contracts in general, have *bilateral* credit risk. Therefore, both the CVA (the credit risk of the counterparty) and DVA (the credit risk of the entity itself) impact the value of the derivative.

$$\text{Value}^{\text{Swap}} = \text{VND} - \text{CVA} + \text{DVA} \tag{3}$$

Typically, the fixed rate on a vanilla interest rate swap is set at inception so that its value is zero. Subsequently, as time passes and market interest rates and credit conditions change, the value will become positive to one party and negative to the other. Moreover, a swap that once was an asset can later become a liability, and vice versa. The VND can be positive or negative; the CVA and DVA are positive amounts—increases in CVA reduce the value of the swap whereas increases in DVA raise it. The examples to follow use a binomial forward rate model for the benchmark interest rate to get the VND for the swap and a credit risk model to get the CVA, DVA, and FVA.

FVA is the newest adjustment to the value of a portfolio of derivatives. JP Morgan Chase first included FVA to its OTC derivatives and structured notes positions in the 4th quarter 2013, "reflecting an industry migration towards incorporating the cost or benefit of unsecured funding into valuations." Its presentation deck further included these bullet points for financial performance of its Corporate and Investment Bank: "Net income of $858mm on revenue of $6.0B; excl. FVA/DVA, net income of $2.1B on revenue of $8.0B" and "FVA loss of $1.5B; DVA loss of $536mm." The FVA can be positive or negative. For example, page 4 of the January 19, 2014 release for Deutsche Bank's Corporate Banking and Securities group states: "Fourth quarter results were also affected by a EUR 110 million charge for Debt Valuation Adjustment (DVA), and a EUR 149 million charge for Credit Valuation Adjustment (CVA), which offset a gain of EUR 83 million for Funding Valuation Adjustment (FVA)."

A net funding cost (FVA > 0) or benefit (FVA < 0) arises when uncollateralized over-the counter (OTC) derivatives are hedged with centrally cleared contracts that require cash collateral. It is a cost when the bank posts collateral and a benefit when it receives the cash. The difference between the interest rate used by the clearinghouse and the bank's cost of funds drives the FVA. The clearinghouse typically pays the "risk-free" rate, which now is the OIS rate, on cash deposits. However, banks obtain their unsecured funds at a spread to LIBOR. Before the crisis, the LIBOR-OIS spread was low and steady at about 8-10 basis points, so FVA reasonably could be deemed immaterial. Since the crisis funding costs and benefits have become significant and some major banks have decided to include FVA in their financial statements. Including funding costs in derivatives valuation is not without controversy, however; see the Hull-White (2012, 2013) for theoretical arguments against FVA.

Section I of this paper describes the construction of a simple binomial term structure model for the benchmark interest rate. This benchmark rate will be the floating reference rate in the interest rate swap contracts covered in the following sections. That same rate is used to discount future cash flows. Therefore, these examples follow the traditional approach to swap valuation in that it abstracts from the post-financial crisis practice of using one set of rates to project future cash flows (e.g., the LIBOR forward curve) and different rates for discounting (e.g., OIS rates). This transition from a *one curve* to a *dual curve* valuation methodology, while important, is not essential to explain CVA, DVA, or FVA. Therefore, simplified examples are used in this presentation.

The following sections illustrate the calculations of the VND, CVA, DVA, and FVA using models to assess the credit risks and funding costs and benefits on an interest rate swap. Section II assesses an uncollateralized interest rate swap between a commercial bank and a corporate counterparty. The specific value for the swap is obtained using VND, CVA, and DVA without regard to how the swap is hedged by the bank. In Section III, funding costs and benefits are introduced that arise from the bank's hedging interest rate risk in the inter-dealer market where cash collateral is used. Two slightly different methods to calculate FVA are illustrated. In this case, the funding benefits exceed the costs; the recognition of FVA < 0 leads to a reported gain in the financial statements. This example, albeit simplified and an abstraction from real contracts, serves to demonstrate the types of valuation models used in practice and how a negative or a positive FVA can be produced for a money-center bank that hedges uncollateralized derivatives with collateralized contracts.

A TERM STRUCTURE MODEL FOR BENCHMARK INTEREST RATES[1]

Exhibit 1 shows the stochastic evolution of the 1-year benchmark interest rate, which in principle could be a government bond yield or an inter-bank rate (such as 12-month LIBOR in the pre-crisis years). The current rate is assumed to be 1.0000%. At the end of the first year, the 1-year rate for the second year will be either 3.6326% or 2.4350%. An important feature of this model is that at each node the odds are 50-50 that the benchmark rate goes up or down—that makes it easy to calculate expected future values. Below each forward rate in parenthesis is the probability of attaining that particular rate. On date 2 at the end of the second year, the possible 1-year forward rates are 5.1111%, 3.4261%, and 2.2966% with probabilities of 0.25, 0.50, and 0.25, respectively.

This forward rate tree is derived from an underlying sequence of hypothetical annual coupon payment, benchmark government bonds. The coupon rates and prices for these bonds are given in Exhibit 2. This is the *par curve* for the benchmark bonds out to five years in that each bond is priced at par value so that the coupon rate equals the yield-to-maturity. Also, there is no accrued interest because the time-to-maturity is an integer.

The next step is to bootstrap the discount factors corresponding to each date. The date-1 discount factor, denoted DF_1, is simply 0.990099 (= 1/1.010000) because the 1-year rate is 1.0000%. The date-2 discount factor is the solution for DF_2 in this equation:

$$100 = (2 * 0.990099) + (102 * DF_2), \qquad DF_2 = 0.960978$$

Note that the future cash flows on the 2-year, 2% bond are 2 and 102 and the current price is 100 (per 100 of par value).

Similarly, the date-3 discount factor is the solution for DF_3, whereby the results from the previous steps are used as inputs—that is the hallmark of bootstrapping. The cash flows on the 3-year, 2.5% bond are 2.5, 2.5, and 102.5 and the current price is again 100.

$$100 = (2.5 * 0.990099) + (2.5 * 0.960978) + (102.5 * DF_3), \qquad DF_3 = 0.928023$$

To generalize, the discount factor for the n^{th} date (DF_n) is:

$$DF_n = \frac{1 - CR_n * \sum_{j=1}^{n-1} DF_j}{1 + CR_n} \tag{4}$$

CR_n is the coupon rate for the n-period bond on the par curve.

These discount factors are used to get the implied forward curve for the 1-year benchmark rate. For the forward rate between date n-1 and date n, denoted $Forward_{n-1,n}$, this formula is used:

$$Forward_{n-1,n} = \frac{DF_{n-1}}{DF_n} - 1 \tag{5}$$

As an example, the 1-year forward rate, three years forward, is 3.7658%. It is the 1-year rate between date 3 and date 4.

$$Forward_{3,4} = \frac{0.928023}{0.894344} - 1 = 0.037658$$

The binomial forward rate tree is designed to branch out around the implied forward rate for each date. There are several assumptions in the calibration process: (1) the forward rates follow a lognormal distribution—that prevents the rates from being negative; (2) rate volatility is assumed to be constant over time, here at 20%—that means the ratio between adjoining rates should be the same; and (3) there are no-arbitrage opportunities in that the rates in the tree correctly value the benchmark bonds.[2]

Exhibit 3 demonstrates the third assumption and illustrates the process of *backward induction* that is used to value securities and derivatives with a binomial tree. Notice that the scheduled cash flows on the 3%, 5-year, annual coupon payment, benchmark bond (per 100 of par value) are placed directly across from each node on the binomial tree. The first coupon payment of 3 on date 1 is across from the date-0 rate of 1.0000%. The final coupon payment and principal redemption for a total of 103 are across from the five possible date-4 forward rates.

Backward induction means that the process starts at maturity on date 5 and works toward the current date 0. If the 1-year rate is 8.0842% at the top of the tree on date 4, the value of the bond is 95.2961 (=

103/1.080842); if the rate is 1.6322% at the bottom of the tree, the bond value is 101.3458 (= 103/1.016322). The values on the earlier dates use the 50-50 odds of upward and downward movement in the rate. In instance, on date 3 if the rate is 4.3694%, the value of bond is 97.2963:

$$\frac{[3+(0.50*97.7053+0.50*99.3898)]}{1.043694}=97.2963$$

The first term in the numerator is the scheduled coupon payment on date 4; the second term is the expected value given the equal probabilities for those two date-4 possibilities. Proceeding backward throughout the tree obtains a value for the bond of 100.0000, demonstrating that the tree is correctly calibrated. [Note: all the examples in this paper are done on a spreadsheet and the rounded values are reported].

VALUING A STANDALONE 4% PAY-FIXED INTEREST RATE SWAP

Assume that two years ago a corporation and a commercial bank entered a 7-year interest rate swap contract. The bank pays a fixed rate of 4% and the corporation pays the 1-year benchmark rate. Net settlement is annual in arrears, meaning that the reference rate is set at the beginning of the year and settlement is at the end of the year. The corporation was motived to create a synthetic floating-rate note, whereby "synthetic" means "made with a derivative". It issued a traditional fixed-rate bond and transformed the obligation into a floating-rate liability by entering the receive-fixed/pay-floating swap. The idea is that this debt structure matches its revenue stream, which happens to be highly correlated to the business cycle and market interest rates.[3] The salient aspect to this swap is that it is uncollateralized—each party bears the credit risk of the other over the lifetime of the contract.

Now, two years later, this 5-year, 4% fixed-rate swap contract needs to be valued for financial reporting. There are several ways to get the VND, the value assuming no default. A classic interpretation of an interest rate swap is that, neglecting counterparty credit risk, its net cash flows are the same as a "long/short" combination of a fixed-rate bond that pays the swap rate and a floating-rate note that pays the reference rate flat. From the perspective of the corporation, the implicit asset is a 5-year, 4% annual coupon payment bond and the implicit liability is a 5-year floating-rate note paying the 1-year benchmark rate. The value of the swap is the difference in the values of the two implicit bonds.

Using the discount factors in Exhibit 2, value of the 5-year, 4% fixed-rate bond is 104.6344 (per 100 of par value).

$$(4 * 0.990099) + (4 * 0.960978) + (4 * 0.928023) + (4 * 0.894344) + (104 * 0.860968)$$
$$= 104.6344.$$

The floating-rate note presumably trades at par value. Remember that credit risk is neglected at this point; in practice floaters can be priced at discounts (or premiums) to par value if the issuer's credit risk has gone up (or down). Therefore, the VND of the swap to the corporation is +4.6344 per 100 of notional principal, the value of the fixed-rate bond (104.6344) less the value of the floater (100). Swaps are a "zero-sum game", so the VND of the swap to the bank is –4.6344.

Another way of getting the VND for the swap is to mark it to market. The contractual fixed rate of 4% is compared to the rate on a "par" or "at-market" swap that has a value of zero. That swap has a fixed rate of 3%, neglecting counterparty credit risk. The 5-year annuity based on the difference between the contractual fixed rate and the market rate is 1 per 100 of notional principal. Discounting that annuity again gives a VND of –4.6344 to the bank, the fixed-rate payer, and +4.6344 to the corporation, the fixed-rate receiver.

$$(1 * 0.990099) + (1 * 0.960978) + (1 * 0.928023) + (1 * 0.894344) + (1 * 0.860968)$$
$$= 4.6344.$$

A third method to obtain the VND is to use the binomial term structure model. The advantage of this approach is that the projected swap values in the tree also are used in the credit risk model to get the CVA and DVA. For now, FVA is neglected. Exhibit 4 shows the binomial tree to get the result that the VND for the 5-year, 4% pay-fixed swap is –4.6344 per 100 of notional principal on the current date 0 from the perspective of the bank. On date 3 when the 1-year benchmark rate is 4.3694%, the value of the swap to the bank is projected to be +0.8289:

$$\frac{[+0.3694+(0.50*1.3461+0.50*-0.3547)]}{1.043694}=0.8289$$

The bank pays the fixed rate of 4% and receives the floating rate of 4.3694%, hence a payment of +0.3694 is owed to the bank by the corporate counterparty at the end of the year on date 4: (0.043694 – 0.0400) * 100 = +0.3694. The expected value of the swap on date 4, given arrival at the date-3 rate of 4.3694%, is (0.50 * 1.3461 + 0.50 * –0.3547). The sum of the settlement payment and the expected swap value is discounted back to date 3 using 4.3694% as the discount rate. Proceeding with backward induction throughout the tree obtains the VND for the swap.

Without regard to counterparty credit risk, the value of the swap is –4.6344 per 100 of notional principal to the bank and +4.6344 to the corporation. The key point is that credit risk is bilateral on these OTC contracts. The CVA measures the expected loss in present value terms resulting from a default by the counterparty (the corporation from the perspective of the bank). The DVA is the expected loss that would be experienced by the corporation if the bank itself were to default. Therefore, it is an imbalance in the relative credit risks that further impacts the value of the transaction.

In general, the CVA/DVA is the sum of the products of four terms for each date:[4]

$$\text{CVA/DVA} = \sum_{t=1}^{T}\left(\text{Expected Exposure}_t\right) * (1-\text{Recovery Rate}_t) * \left(\text{Default Probability}_t\right) * \left(\text{Discount Factor}_t\right) \tag{6}$$

The (1 – Recovery Rate_t) term is known as the *loss severity*. The product of the expected exposure and the loss severity is called the expected loss given default. That amount times the probability of default is the expected loss. The discount factors for the T dates are bootstrapped from the underlying benchmark bonds as described in the previous section. The expected exposure is a key element in the CVA/DVA calculation. It is the expected settlement payment and value of the interest rate swap on each future date if it were risk-free—this is where the binomial tree model for VND and the probabilities of attaining particular values at the various nodes come into play.

The key credit risk parameters are the assumed probabilities of default and the recovery rate. These are exogenous to the model, as if they are determined by separate credit analysts and then given to the swap trading desk at the commercial bank (or its internal auditor) for use in valuation. A limitation of this simple valuation model is that these parameters are assumed to be independent of the level of benchmark interest rates for each future date. In reality, market rates and the business cycle are positively correlated by means of monetary policy. When the economy is strong—and presumably the probabilities of default by both the corporation and the bank are lower—interest rates tend to be higher because the central bank is tightening the supply of money and credit. When the economy is weak, and default probabilities are higher, expansionary monetary policy lowers benchmark rates. Of course, this correlation varies by the nature of the underlying business.

For this example, the corporation is simply assumed to have a default probability of 2.50% and a recovery rate of 40% for each year.[5] This suggests that the corporation has a quality rating in the B/B+ range. The commercial bank is assumed to have a default probability of 0.50% and a recovery rate of 10%

for each year, indicating a "Lehman-like" financial institution for which the probability of default is perceived to be low (nowadays it is a SIFI, a "systemically important financial institution") but the loss severity would be high if that event happens. Unsecured creditors on derivatives stand behind depositors in the priority of claim.

The CVA and DVA are calculated in Exhibit 5. The key element is the expected exposure for each year. Note that the model assumes that default will not occur on date 0. Also, this model assesses the credit risk of the swap in isolation and not as part of a portfolio that would trigger *closeout netting* provisions. If the corporation defaults, all of its derivatives with the commercial bank would be aggregated and the exposure is the net positive value. Therefore, negative values in the binomial tree can be used to offset positive exposures on other contracts with the counterparty. In this simple model of a single swap transaction, the negative payments and swap values revealed in the binomial tree are converted to zeros. Only positive payments and values factor into the calculation of credit risk, i.e., CVA and DVA.

The expected exposure facing the bank on each date is the probability-weighted average of the (positive) settlement payments and swap values for each date taken from Exhibit 4. For example, the expected exposure on date 3 is 1.1848 (per 100 of notional principal).

$$[0.25*1.1111+0.50*0+0.25*0]\ + \\ [0.125*4.7699+0.375*0.8289+0.375*0+0.125*0]=1.1848$$

The first term in brackets is the expected settlement payment on date 3 based on the realized date-2 rates and the probabilities of rates attaining each node in the tree. The negative payments (–0.5739 and (–1.7034) convert to zero. The second term in brackets is the expected swap value on date 3 based on the date-3 rates and their probabilities, again replacing the negative values with zeroes. Note that the probabilities of arriving at each node are shown in Exhibit 1. The CVA for each date is the product of the expected exposure, the loss severity, the probability of default, and the discount factor. As of date 0, the credit risk of the corporate counterparty is 0.0583 (per 100 of notional principal).

The DVA is calculated in the same manner, using the assumed default probability of 0.50% and recovery rate of 10% that apply to the commercial bank. The expected exposure to the corporation arising from default by the bank is 1.3169 on date 4.

$$[0.125*0+0.375*0+0.375*1.0711+0.125*2.0367] \\ +\ [0.0625*0+0.25*0+0.375*0.3547+0.25*1.5279+0.0625*2.3298]=1.3169$$

Exhibit 4 shows the swap from the perspective of the bank. From the perspective of the corporate fixed-rate receiver, all of the signs are reversed. The negative payments and swap values that are replaced with zeros occur in the top half of the tree. The DVA turns out to be 0.0503. Using equation (3), the *fair value* of the pay-fixed swap to the commercial bank is –4.6424 per 100 of notional principal:

$$\text{Value}^{\text{SWAP}} = \text{VND} - \text{CVA} + \text{DVA} = -4.6344 - 0.0583 + 0.0503 = -4.6424$$

"Fair value" is an accounting term—in SFAS 157 it is defined as "the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date." Essentially, it is an *exit price* determined either by observable inputs (i.e., *mark-to-market* valuation) or assumed inputs (*mark-to-model*).

This numerical example of valuation illustrates an important aspect of counterparty credit risk on an interest rate swap. It is bilateral in that on an uncollateralized swap both counterparties factor the potential loss due to the other's default into the value of the contract. The weaker the credit risk parameters of the counterparty, meaning a higher probability of default and/or a lower recovery rate given default, the lower is the fair value of the swap. Also, the stronger the credit quality of the counterparty (as well as the weaker the party itself), the higher is the value.

The relative amount of counterparty credit risk, i.e., the CVA and DVA, is a function of three factors: (1) the difference in credit risk parameters—the probability of default and the recovery rate of each party to the contract, (2) the sign of the VND as that is a major element in the expected exposure, and (3) the shape of the benchmark yield curve. In this example, the effect of the credit risk parameters is not obvious because, by assumption, the corporation has a higher probability of default than the commercial bank (2.50% vs. 0.50%) but also a lower loss severity (60% vs. 90%). In this example, the VND is +4.6344 to the corporation. Even if the credit risk parameters for the counterparties were the same, the corporation would rightfully be more concerned about potential default than the bank.

An implication of the third factor, the shape of the benchmark yield curve, is that even if the counterparties have the same probabilities of default and recovery rates and the VND for the swap is zero, the CVA and DVA will not be equal. When the yield curve is upwardly sloped, the expected exposure facing the fixed-rate payer is relatively "back-loaded" because there are more positive values and payments in the second half of the swap's lifetime. On the other hand, the expected exposure to the fixed-rate receiver is "front-loaded". As an example, a 5-year, 3% pay-fixed swap on the 1-year benchmark rate has a VND of zero. If the parties both have a default probability of 1% and a recovery rate of 40%, the CVA and DVA are 0.0608 and 0.0231, respectively, from the perspective of the fixed-rate payer. That results in a negative swap value. This arises because the underlying benchmark yield curve in Exhibit 2 slopes upward.

VALUING THE COMBINATION OF THE 4% PAY-FIXED INTEREST RATE SWAP AND THE HEDGE SWAP[6]

Funding costs arise from hedging an uncollateralized derivative with a comparable contract that entails posting cash collateral that earns an interest rate lower than the bank's cost of borrowed funds. The bank posts collateral on the hedge swap to cover negative values and payments. A funding benefit results from receiving cash as collateral on the hedge swap and paying a lower interest rate than the cost of funds. The FVA is the net difference between the funding cost and benefit:

$$\text{FVA} = \text{Funding Cost} - \text{Funding Benefit} \tag{7}$$

To see how funding costs and benefits arise, suppose that the commercial bank that has on its books the 5-year, 4% pay-fixed swap with the corporate counterparty in Section II hedged its interest rate risk at inception with a 4.05% receive-fixed swap in the inter-dealer market. Both swaps are tied to the 1-year benchmark rate, make net settlement payments in arrears, and have the same notional principal. The combination of receiving the floating reference rate from the corporation and paying that same rate to the dealer eliminates exposure to subsequent changes in the 1-year benchmark rate. This is illustrated in Exhibit 6. The difference in the fixed rates establishes that the profit to the bank on the hedged transaction is an annuity of five basis points per year assuming no default.

Two years before, when the 4% swap with the corporation and the 4.05% hedge swap were initiated, each had a tenor of seven years and an initial value of zero. In fact, the 4% pay-fixed rate would have been set based on the 4.05% rate on the hedge swap. A principle of OTC derivatives pricing is that the rate given a customer is a markup or markdown from the rate on the hedge product or strategy. That is, the commercial bank first identifies how the credit and interest rate risks on the OTC derivative will be managed and then sets the rate to the customer to cover the costs and risk of hedging, as well as some target profit. The bank started with 4.05% on the receive-fixed swap in the inter-dealer market and then used a credit risk model to get the CVA and DVA. The pay-fixed rate was chosen to be 4.00% because at that rate the fair value of the swap was zero at inception.

While the interest rate risk on the OTC swap with the corporation is hedged, the credit risk is not. The bank remains at risk to changes in the CVA and DVA, in particular, that the expected loss given default by the counterparty goes up. Some money-center banks in recent years have centralized CVA, thereby aggregating the credit risk exposures with counterparties arising on derivatives as well as loan contracts. Then the CVA trading desk can choose to hedge some or all of that risk using credit default swaps. The exposure to the bank's own credit risk, i.e., to lower DVA, is much more difficult to hedge. Presumably, this risk is priced into the fixed rate on the swap.

In Section II the fair value of the pay-fixed swap to the bank is determined to be –4.6424 per 100 of notional principal. That liability is offset by the increased value of the receive-fixed hedge swap as market rates have come down over the two years. Exhibit 7 shows that the VND for the 5-year, 4.05% receive-fixed hedge swap with a dealer is +4.8661 per 100 of notional principal. The salient feature to this inter-dealer swap is that it is fully collateralized, presumably with cash that earns the benchmark interest rate. Collateralization in principle renders the swap to be free of default risk; therefore by assumption CVA = DVA = 0. The hedge swap's fair value is +4.8661 before consideration of the funding costs and benefits associated with collateralization. This assumption about the absence of credit risk implies that some aspects of actual collateral agreements are neglected—for instance, thresholds that indicate when collateral needs to be posted, minimum transfer amounts, and timeframes for the delivery of the cash.

If these two swaps comprise the commercial bank's entire derivatives portfolio, the net value is +0.2237 per 100 of notional principal: –4.6424 + 4.8661 = +0.2237. Of course, banks that are active in the OTC derivatives market have multitudes of contracts. Often the swaps provide "internal" hedges to interest rate risk, for instance, when pay-fixed swaps are offset with receive-fixed swaps with other end-users. Then the bank only uses the inter-dealer market to hedge the residual interest rate risk.

Now suppose that the commercial bank chooses to introduce the net funding costs and benefits into its valuation of the "two-swaps" derivatives portfolio. These costs and benefits arise fundamentally from the spread between the bank's 1-year cost of funds on the money market and the 1-year benchmark rate. However, there are different ways to calculate the FVA for the collateralized swap—and the financial statements of the major money-center banks that have adopted FVA in recent years do not disclose their methodology. Each of two methods illustrated here rely on the same set of assumptions about the credit risk parameters for the bank.

Exhibit 8 shows the tables for the first method to calculate the FVA on the 4.05% hedge swap. These tables parallel those used to get the CVA and DVA in Section II. Some arbitrary assumptions need to be made in this simple model. Cash collateral is posted once a year to cover the net settlement payment that is owed to the dealer that is the counterparty to the hedge swap and to cover a negative value to the contact on that date. Similarly, cash is received from dealer when the net settlement payment is owed to the bank and to cover a positive swap value on that date. The expected amounts of collateral to be posted and received for each date are based on the binomial tree in Exhibit 7 for the 4.05% receive-fixed swap in the inter-dealer market.

The upper table in Exhibit 8 shows the expected funding costs to be 0.0137 per 100 of par value; the lower table shows the expected funding benefit to be 0.0697. The FVA is –0.0560, indicating an FVA gain to the commercial bank: 0.0137 – 0.0697 = –0.0560. The key calculations are the expected cash collateral that is posted or received each year. Given that the date-0 VND is +4.8661, the bank receives that amount in cash from the dealer and pays only the 1-year benchmark rate for use of those funds. The produces a date-1 benefit in the lower table and no cost in the upper table. The amount of the benefit is the "haircut" that does not have to be paid to acquire the cash. In the money market, the bank pays the benchmark rate plus a credit spread that depends on the probability that the bank defaults and the loss severity. That benefit is 0.0217 per 100 of notional principal in present value terms: 4.8661 * 90% * 0.50% * 0.990099 = 0.0217. Even though there are only two derivatives in the portfolio, it is assumed that the bank benefits when obtaining cash for use in other operations and having to pay only the benchmark rate to obtain that cash.

The date-4 expected costs and benefits depend on the signs of the settlement payment and swap values on date 3 in Exhibit 7. As with the calculation of CVA and DVA, the probabilities of attaining those amounts are used to get the expected collateral flows. The expected amount of cash to be posted on date 3 is 1.1258 per 100 of notional principal:

$$[0.25*1.0611+0.50*0+0.25*0]$$
$$+\ [0.125*4.6790+0.375*0.7351+0.375*0+0.125*0]=1.1258$$

Those are the negative numbers in Exhibit 7 for date 3. The positive numbers revert to zero because those entail the receipt of cash collateral, resulting in a benefit rather than a cost. Similarly, the expected amount of cash to be received on date 3 is 2.0177:

$$[0.25*0+0.50*0.6239+0.25*1.7534]$$
$$+\ [0.125*0+0.375*0+0.375*2.0509+0.125*3.9863]=2.0177$$

These expected collateral flows generate the expected funding cost of 0.0045 and a benefit of 0.0081 for date 4. The benefit arises from receiving cash and not having to pay the "haircut"—the bank is effectively borrowing funds at a below-market interest rate. The cost is incurred because cash to meet the collateral requirement must be acquired in the money market at the market rate for its credit standing. The credit spread over the benchmark rate that the bank pays lenders is passed on to the swap trading desk in the form of the FVA. The reality that the bank does not issue debt at the benchmark rate is a cost to the business of derivatives market-making.

The FVA for the 5-year, collateralized, 4.05% received-fixed hedge swap is a present value of –0.0560, indicating an FVA gain to the commercial bank because in this case the expected funding benefit exceeds the costs. Notice that formulated in this manner the FVA is a hybrid of the CVA and DVA. The pattern for expected collateral to be posted each date that is used to get the expected funding cost, i.e., the upper table in Exhibit 8, is similar to the pattern for expected exposure to default by the corporate counterparty used to get the CVA in Exhibit 5. The pattern for the receipt of cash collateral (the lower table in Exhibit 8) is similar to the expected exposure to default by the bank used to get the DVA in Exhibit 5. The same credit risk parameters associated with the bank—the assumed probability of default and the recovery rate—are used to get both the FVA and the DVA.

The inclusion of the FVA modifies equation (3) for the value of the hedge swap:

$$\text{Value}^{SWAP} = \text{VND} - \text{CVA} + \text{DVA} - \text{FVA} \tag{8}$$

The 5-year, collateralized, 4.05% receive-fixed swap has a VND = +4.8661, CVA = 0, DVA = 0, and FVA = –0.0560. Overall, the value of the hedge swap is +4.9221.

$$\text{Value}^{SWAP} = +4.8661 - 0 + 0 - (-0.0560) = +4.9221$$

Recognizing the FVA gain on the hedge swap, the value of the derivatives portfolio is raised from 0.2273 (= –4.6424 + 4.8661) to 0.2793 (= –4.6424 + 4.9221). Notice that the value of the swap with the corporate counterparty is unchanged. The net funding cost or benefit applies to the hedge swap.

The second method to calculate FVA is to project the 1-year cost of funds in the money market using the benchmark rate for each node in the tree and the credit risk parameters. Given the assumptions about the bank's probability of default (PD) for each year and the recovery rate (RR), the market rate (MR) for the bank's 1-year money market debt relative to the benchmark rate (BR) can be estimated as follows:

$$(1 + \text{BR}) = [(1 - \text{PD}) * (1 + \text{MR})] + [\text{PD} * (1 + \text{MR}) * \text{RR}] \tag{8}$$

The left-side of the equation is the return per dollar invested in the 1-year risk-free benchmark security. The right-side is the weighted average return on buying the bank's 1-year debt liability that pays the market rate, using the probability of default and no-default as the weights. Notice that as PD approaches zero and as RR approaches one, MR equals BR and the bank's credit spread becomes zero. Rearranging equation (8) gives an equation for the market rate given the other variables.

$$\mathrm{MR} = \frac{\mathrm{BR} + \left[\mathrm{PD} * (1 - \mathrm{RR})\right]}{1 - \left[\mathrm{PD} * (1 - \mathrm{RR})\right]} \tag{9}$$

This equation assumes risk-neutrality on the part of money market investors in that there is no term for risk aversion.

Exhibit 9 shows the binomial tree for the commercial bank's 1-year cost of funds based on its credit risk parameters: PD = 0.50% and RR = 10%. For instance, on date 1 when the benchmark rate is 3.6326% in Exhibit 1, the bank's market rate is 4.1011% calculated using equation (9).

$$\mathrm{MR} = \frac{0.036326 + \left[0.0050 * (1 - 0.10)\right]}{1 - \left[0.0050 * (1 - 0.10)\right]} = 0.041011$$

On date 4 when the benchmark rate is 8.0842%, the bank's market for 1-year funds is 8.5728%.

$$\mathrm{MR} = \frac{0.080842 + \left[0.0050 * (1 - 0.10)\right]}{1 - \left[0.0050 * (1 - 0.10)\right]} = 0.085728$$

Notice that the credit spread increases with the level of the benchmark rate. It is 46.85 basis points in the first example (4.1011% - 3.6326% = 0.4685%) and 48.86 basis points in the second (8.5728% - 8.0842% = 0.4886%).

Exhibit 10 shows the tables used to calculate the FVA on the 4.05% hedge swap for the second method. The key calculations are the expected cash collateral that is posted or received each year. Given that the date-0 VND is +4.8661, the bank receives that amount in cash from the dealer and pays only the 1-year benchmark interest rate. That produces a date-1 benefit in the lower table and no cost in the upper table. The amount of the benefit is the 0.0222 [= 4.8661 * (1.4566% – 1.0000%)] as of date 1 at the end of the year.

The negative payments and values in Exhibit 7 signal funding costs. For example, the projected value of the swap is –0.7289 on date 1 when the benchmark rate is 3.6326%. The expected funding cost for the year is 0.0017 [= 0.5 * 0.7289 * (4.1011% – 3.6326%)] as of date 2. It is the 50% probability of attaining that (negative) swap value times the amount of the value times the projected credit spread, which is the difference between the market rate and the benchmark rate.

Likewise, the positive payments and values for each date lead to funding benefits. The expected benefit for the second year is 0.0245 [= 0.5 * 4.4585 * (2.8980% – 2.4350%)] + 3.0500 * ((0.5 * (4.1011% - 3.6326%) + 0.5 * (2.8980% - 2.4350%))]. The first term in brackets is the 50% probability of attaining that (positive) value times the amount of the value times the credit spread for the benchmark rate having gone "down" after the first year. The second term is the known settlement receipt of 3.0500 times the expected value for the credit spread. The odds are 50-50 that the benchmark rate goes up from 1.0000% to 3.6326% and "down" to 2.4350%.

The expected funding costs and benefits for the third year are calculated in the same manner. The expected cost for the third year as of Date 3 is 0.0043 [= 0.25 * 3.5849 * (5.5862% – 5.1111%)]. The expected benefit is more complicated because there are two possible positive settlement payments and two positive swap values. It is 0.0130 [= 0.5 * 1.2393 * (3.8936% – 3.4261%) + 0.25 * 4.6648 * (2.7590% – 2.2966%) + 0.5 * 0.4174 * ((0.5 * (5.5862% – 5.1111%) + 0.5 * (3.8936% – 3.4261%)) + 0.5 * 1.6150 * ((0.5 * (3.8936% – 3.4261%) + 0.5 * (2.7590% – 2.2966%))]. The first two terms deal with the swap values and the last two with the settlement payments.

Using the second method, the overall expected funding cost is 0.0145 (per 100 of notional principal) and the funding benefit is 0.0715. Combined, the FVA is –0.0570, indicating a net funding benefit to the bank slightly higher than the result for the first method. The benefit arises because interest rates have decrease since the swap was initiated. The uncollateralized payer swap with the corporate counterparty is "out of the money" to the bank as it has negative fair value. However, the collateralized receiver swap with the dealer is "in the money". The receipt of cash collateral provides a benefit because the bank can use those funds in its operations and pay only the benchmark rate instead of its own market rate that reflects its credit risk parameters.

The small difference in the FVA results from the calculation methodology. In the first method, the "haircut" is based on the expected amount of cash collateral to be posted or received. That amount is the assumed principal on the 1-year security. In the second method, the market rate is projected, given the credit risk parameters, such that a money market investor's expected return matches that on the benchmark security, including both principal and interest. Including the interest explains why the second method is slightly higher. There are obviously many other assumptions that go into implementing FVA in practice. For instance, here it is assumed that the swap remains on the books of the bank for the remaining five years—and incurs five-years worth of funding benefits and costs. In reality, many swaps are terminated early. Therefore, the bank could assume some pattern of decline in the notional principal on the overall derivatives portfolio.

In the example, the FVA gain to the bank arises because the VND on the underlying pay-fixed swap with the corporate counterparty is negative. Market interest rates have come down since inception so the gain on received-fixed hedge swap offsets that negative value. This is all reversed if the legs to the underlying swap are opposite. If the commercial bank had entered a 4% receive-fixed swap with the corporation, the VND would be positive. Then the pay-fixed hedge swap would require posting cash collateral. The recognition of the net funding cost results in an FVA loss.

The key point is that the positions the bank has in collateralized swaps—and to funding costs and benefits—typically will be exogenously determined by demand for derivatives from its customers. Of course, the bank will have some influence on the types of derivatives that it emphasizes in its marketing programs but to a large extent the composition of its holdings is customer-driven. Therefore, an analyst needs to be cognizant of how and why funding valuation adjustments are made before assessing an FVA gain or loss.

CONCLUSIONS

This paper illustrates modern derivatives valuation in which counterparty credit risk is front and center. Separate models are used to determine the value assuming no default (VND) and the adjustments needed for bilateral credit risk that is inherent in interest rate swaps (CVA and DVA). These model are necessarily complex and mathematical. Nevertheless, users of the outputs of the models should have some understanding of the nature of the models and the key inputs and assumptions.

Funding valuation adjustments (FVA) are now being introduced in the financial statements of money-center banks to recognize the impacts of hedging unsecured derivatives with collateralized contracts and the difference between the "risk-free" rates used on cash collateral and the bank's own cost of funds. These adjustments can be significant, especially in the quarter when the adjustment is first registered, as evidenced by JP Morgan Chase's FVA loss of USD 1.5 billion in 1993 and Bank of America's loss of $497 million in 1994. It is important for users of the statements to understand that these are accounting ramifications of derivatives market-making in the post-financial-crisis world.

NOTE: The author thanks Sunjoon Park for careful editing and checking calculations in the original version and Zilong Zheng for the latest version. The author is responsible for remaining misstatements and errors.

EXHIBIT 1
BINOMIAL FORWARD RATE TREE FOR THE 1-YEAR BENCHMARK INTEREST RATE

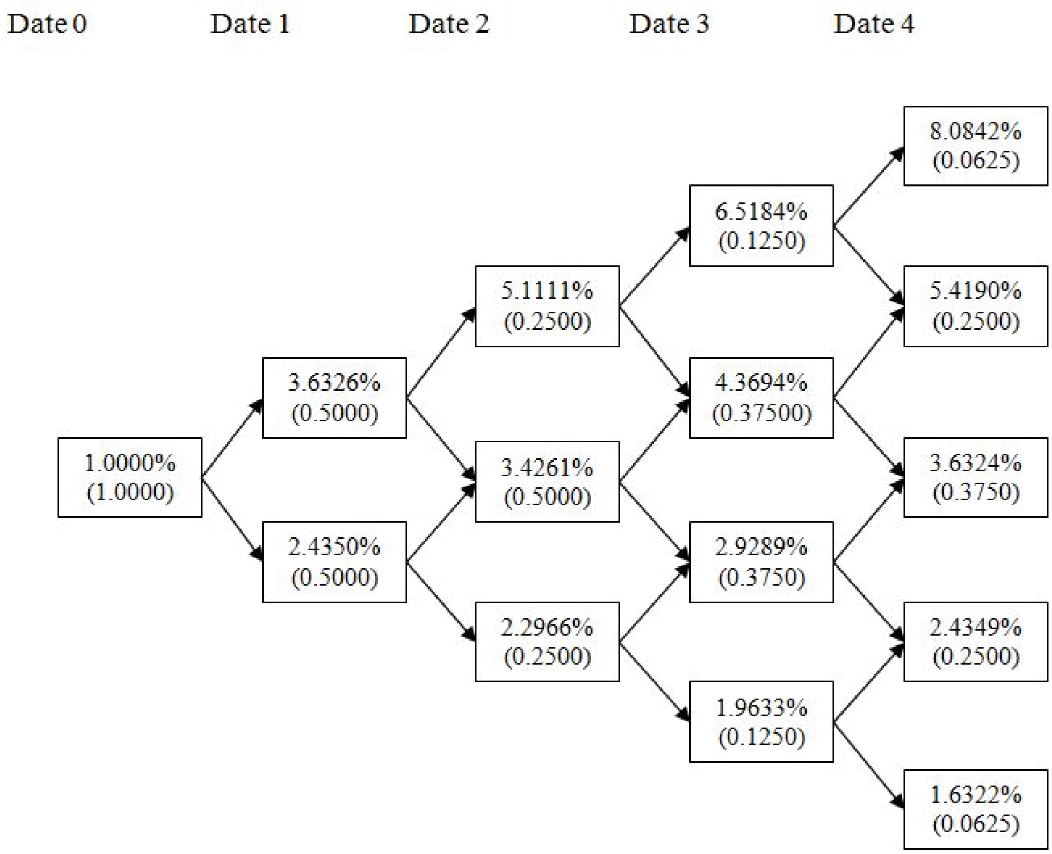

EXHIBIT 2
UNDERLYING RISK-FREE BENCHMARK COUPON RATES, PRICES, AND DISCOUNT FACTORS

Dates	Coupon Rates	Prices	Discount Factors
1	1.00%	100.000	0.990099
2	2.00%	100.000	0.960978
3	2.50%	100.000	0.928023
4	2.80%	100.000	0.894344
5	3.00%	100.000	0.860968

EXHIBIT 3
VALUATION OF THE 3%, 5-YEAR, ANNUAL COUPON PAYMENT BENCHMARK BOND

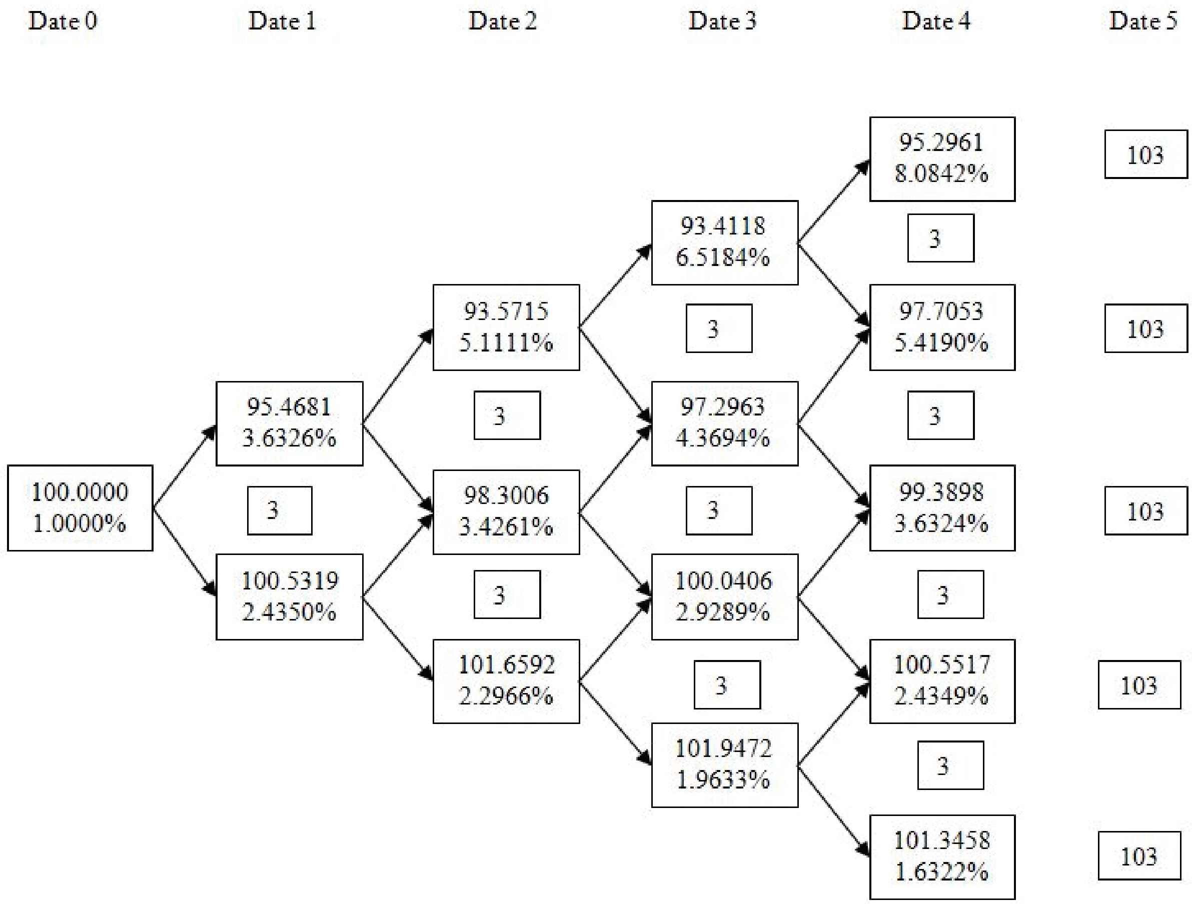

EXHIBIT 4
VALUATION OF THE 4%, 5-YEAR, PAY-FIXED INTEREST RATE SWAP ASSUMING NO DEFAULT

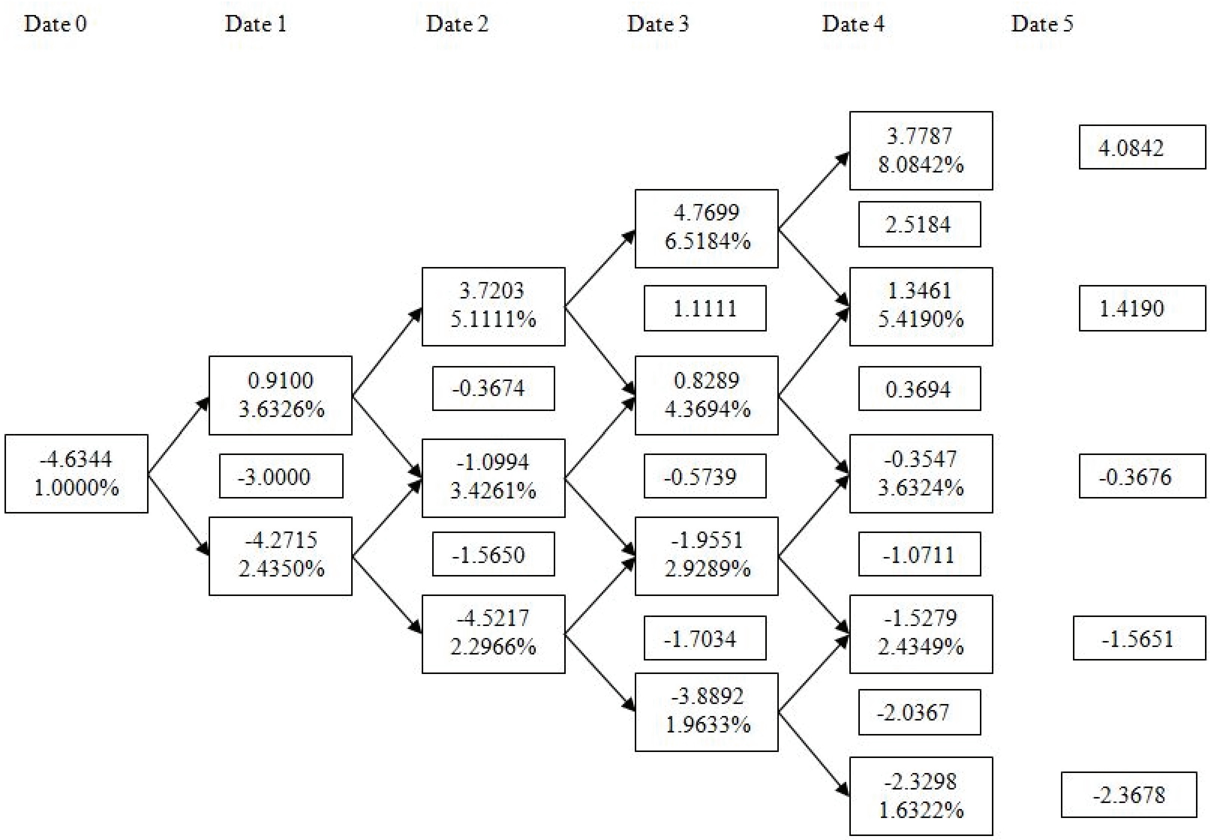

EXHIBIT 5
CVA AND DVA CALCULATIONS ON THE 4%, 5-YEAR, PAY-FIXED INTEREST RATE SWAP

Credit Risk of the Fixed-Rate Receiver—the Corporate Counterparty

	Expected Exposure	Loss Severity	Probability of Default	Discount Factor	CVA
Date 0					
Date 1	0.4550	60%	2.50%	0.990099	0.0068
Date 2	0.9301	60%	2.50%	0.960978	0.0134
Date 3	1.1848	60%	2.50%	0.928023	0.0165
Date 4	1.0260	60%	2.50%	0.894344	0.0138
Date 5	0.6100	60%	2.50%	0.860968	0.0079
					0.0583

Credit Risk of the Fixed-Rate Payer—the Commercial Bank

	Expected Exposure	Loss Severity	Probability of Default	Discount Factor	DVA
Date 0					
Date 1	5.1358	90%	0.50%	0.990099	0.0229
Date 2	2.6463	90%	0.50%	0.960978	0.0114
Date 3	1.9321	90%	0.50%	0.928023	0.0081
Date 4	1.3169	90%	0.50%	0.894344	0.0053
Date 5	0.6771	90%	0.50%	0.860968	0.0026
					0.0503

EXHIBIT 6
THE 4% PAY-FIXED SWAP WITH THE CORPORATION HEDGED WITH THE 4.05% RECEIVED-FIXED SWAP IN THE INTER-DEALER MARKET

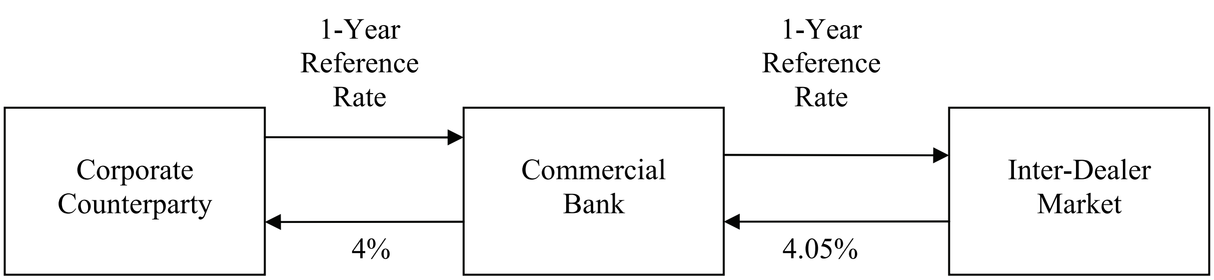

EXHIBIT 7
VALUATION OF A 4.05%, 5-YEAR, RECEIVE-FIXED INTEREST RATE SWAP ASSUMING NO DEFAULT

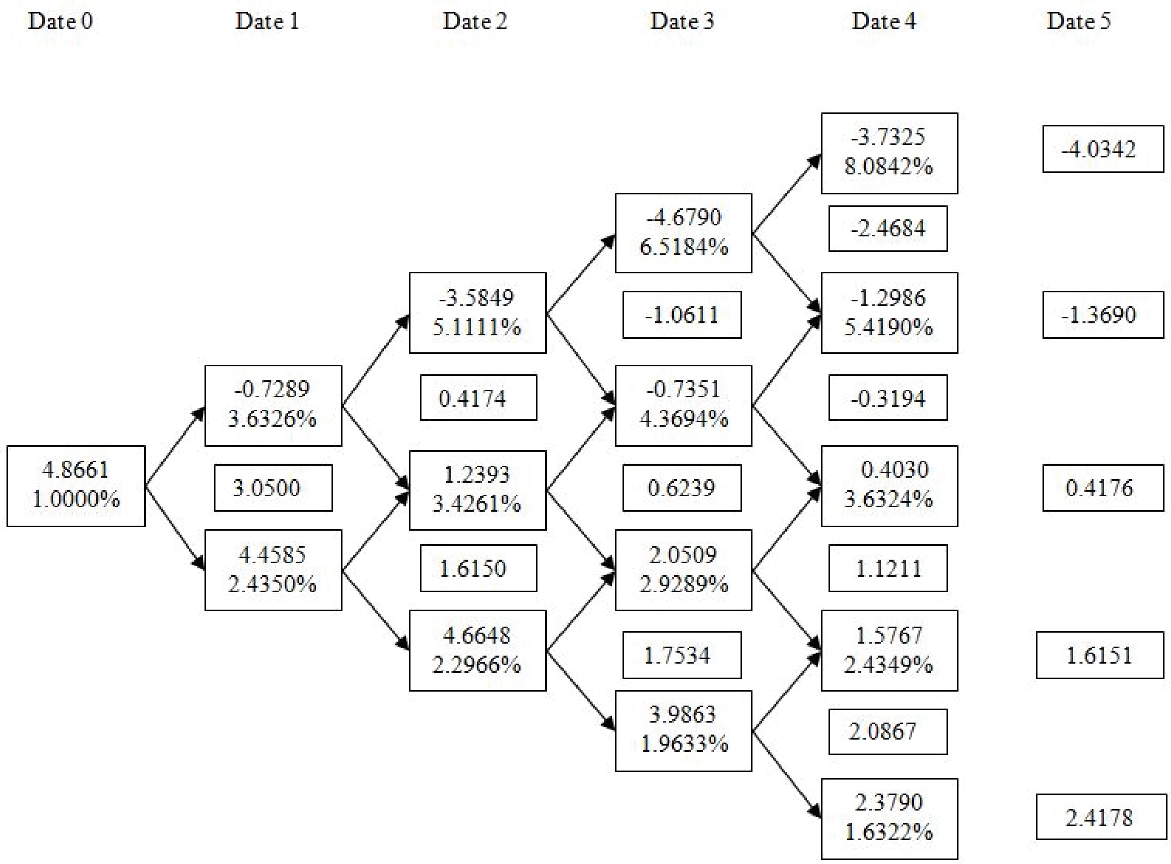

EXHIBIT 8
FIRST METHOD TO CALCULATE THE FUNDING COSTS AND BENEFITS

Funding Costs

	Expected Posting of Cash Collateral	Loss Severity	Probability of Default	Discount Factor	Expected Funding Costs per Year
Date 1	0	90%	0.50%	0.990099	0.0000
Date 2	0.3645	90%	0.50%	0.960978	0.0016
Date 3	0.8962	90%	0.50%	0.928023	0.0037
Date 4	1.1258	90%	0.50%	0.894344	0.0045
Date 5	0.9863	90%	0.50%	0.860968	0.0038
					0.0137

Funding Benefits

	Expected Receipt of Cash Collateral	Loss Severity	Probability of Default	Discount Factor	Expected Funding Benefit per Year
Date 1	4.8661	90%	0.50%	0.990099	0.0217
Date 2	5.2793	90%	0.50%	0.960978	0.0228
Date 3	2.8021	90%	0.50%	0.928023	0.0117
Date 4	2.0177	90%	0.50%	0.894344	0.0081
Date 5	1.3752	90%	0.50%	0.860968	0.0053
					0.0697

FVA = Funding Costs – Funding Benefits = 0.0137 – 0.0697 = –0.0560

EXHIBIT 9
BINOMIAL FORWARD RATE TREE FOR THE COMMERCIAL BANK'S 1-YEAR MONEY MARKET RATE

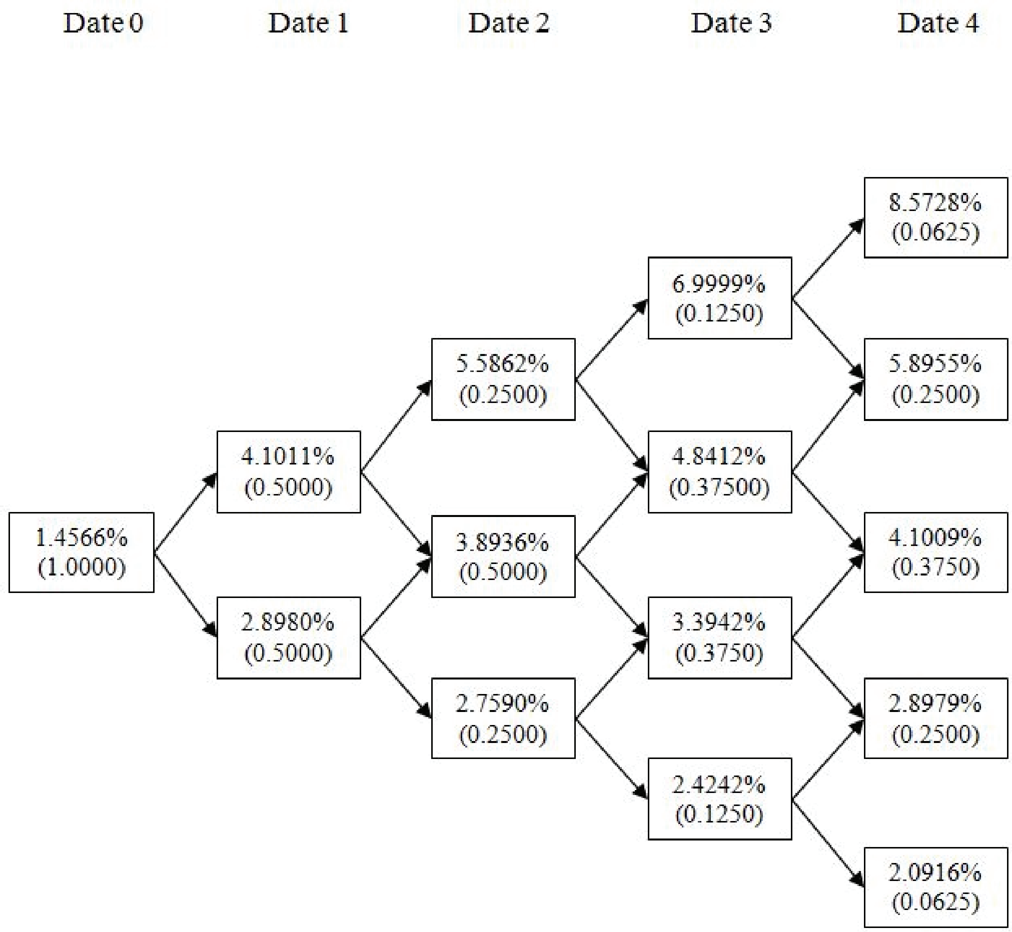

EXHIBIT 10
SECOND METHOD TO CALCULATE THE FUNDING COSTS AND BENEFITS

Funding Costs

Date	Expected Funding Cost Per Year	Benchmark Discount Factor	PV of Funding Costs
1	0.0000	0.990099	0.0000
2	0.0017	0.960978	0.0016
3	0.0043	0.928023	0.0040
4	0.0054	0.894344	0.0048
5	0.0047	0.860968	0.0041
			0.0145

Funding Benefits

Date	Expected Funding Benefit Per Year	Benchmark Discount Factor	PV of Funding Benefit
1	0.0222	0.990099	0.0220
2	0.0245	0.960978	0.0236
3	0.0130	0.928023	0.0121
4	0.0094	0.894344	0.0084
5	0.0064	0.860968	0.0055
			0.0715

FVA = Funding Costs – Funding Benefits = 0.0145 – 0.0715 = –0.0570

ENDNOTES

1. The binomial tree derived in this section is a discrete version of the Kalotay-Williams-Fabozzi (KWF) model (1993). It has been used in the CFA® curriculum since 2000; hence it is familiar to many finance professionals and is very suitable for pedagogy. A key difference is that the model herein is for the benchmark interest rate whereas the original KWF model is for the bond issuer's own cost of funds including its credit spread over the benchmark.
2. This a classroom "artisanal" model that students can build themselves on a spreadsheet, so rounding the rates to four digits introduces some insignificant modeling error. For instance, given 20% volatility and the assumed lognormal distribution, the relationship between adjoining rates for each date should be a multiple of 1.491825, but in this tree it close to but not always equal to that number.
3. See Adams and Smith (2013) for further motivations for this type of asset-driven liability structure.
4. This expression for CVA is based on an example in chapter 7 of Gregory (2010). This book provides comprehensive coverage of counterparty credit risk.
5. Technically, the probability of default should be conditional on no prior default. That is, it is the conditional default probability, also known as the hazard rate. Using the same probability for each year simplifies the presentation.
6. This example is based on the discussion of FVA in a publication by KPMG (2013).

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